LH 2

(1)

$$A = \begin{bmatrix} \cos(-\alpha), & -\sin(-\alpha) \\ \sin(-\alpha), & \cos(-\alpha) \end{bmatrix} \qquad B = \begin{bmatrix} \cos\alpha, & -\sin\alpha \\ \sin\alpha, & \cos\alpha \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos\alpha, & \sin\alpha \\ -\sin\alpha, & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha, & -\sin\alpha \\ \sin\alpha, & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{\cos\alpha\cdot\cos\alpha + \sin\alpha\cdot\sin\alpha}_{1}, & \underbrace{\cos\alpha\cdot(-\sin\alpha) + \sin\alpha\cdot\cos\alpha}_{0} \\ \underbrace{-\sin\alpha\cdot\cos\alpha + \cos\alpha\cdot\sin\alpha}_{0}, & \underbrace{-\sin\alpha\cdot(-\sin\alpha) + \cos\alpha\cdot\cos\alpha}_{1} \end{bmatrix}$$

$$= I$$

ehto $AB = BA = I$

$$BA = \begin{bmatrix} \cos\alpha, & -\sin\alpha \\ \sin\alpha, & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{\cos\alpha\cdot\cos\alpha + (-\sin\alpha)\cdot(-\sin\alpha)}_{1}, & \underbrace{\cos\alpha\cdot\sin\alpha + (-\sin\alpha)\cdot\cos\alpha}_{0} \\ \underbrace{\sin\alpha\cdot\cos\alpha + \cos\alpha\cdot(-\sin\alpha)}_{0}, & \underbrace{\sin\alpha\cdot\sin\alpha + \cos\alpha\cdot\cos\alpha}_{1} \end{bmatrix}$$

(2)

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad P^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

a)

$$P^T P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot 0 + 1\cdot 1 + 0\cdot 0, & 0\cdot 0 + 1\cdot 0 + 0\cdot 1, & 0\cdot 1 + 1\cdot 0 + 0\cdot 0 \\ 0\cdot 0 + 0\cdot 1 + 1\cdot 0, & 0\cdot 0 + 0\cdot 0 + 1\cdot 1, & 0\cdot 1 + 0\cdot 0 + 1\cdot 0 \\ 1\cdot 0 + 0\cdot 1 + 0\cdot 0, & 1\cdot 0 + 0\cdot 0 + 0\cdot 1, & 1\cdot 1 + 0\cdot 0 + 0\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

b) $P^{-1} = P^T$

koska $P^T P = P^T P = I$

$$P P^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot 0 + 0\cdot 0 + 1\cdot 1, & 0\cdot 1 + 0\cdot 0 + 1\cdot 0, & 0\cdot 0 + 0\cdot 1 + 1\cdot 0 \\ 1\cdot 0 + 0\cdot 0 + 0\cdot 0, & 1\cdot 1 + 0\cdot 0 + 0\cdot 0, & 1\cdot 0 + 0\cdot 1 + 0\cdot 0 \\ 0\cdot 0 + 1\cdot 0 + 0\cdot 1, & 0\cdot 1 + 1\cdot 0 + 0\cdot 0, & 0\cdot 0 + 1\cdot 1 + 0\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Induktiolla:

PA: $\begin{bmatrix}1&0\\1&1\end{bmatrix}^1=\begin{bmatrix}1&0\\1&1\end{bmatrix}$ ok.

IO: $\begin{bmatrix}1&0\\1&1\end{bmatrix}^n=\begin{bmatrix}1&0\\n&1\end{bmatrix}$

$$\begin{bmatrix}1&0\\1&1\end{bmatrix}^{n+1}=\begin{bmatrix}1&0\\1&1\end{bmatrix}\cdot\begin{bmatrix}1&0\\1&1\end{bmatrix}^n\overset{IO}{=}\begin{bmatrix}1&0\\1&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\n+1&1\end{bmatrix}$$

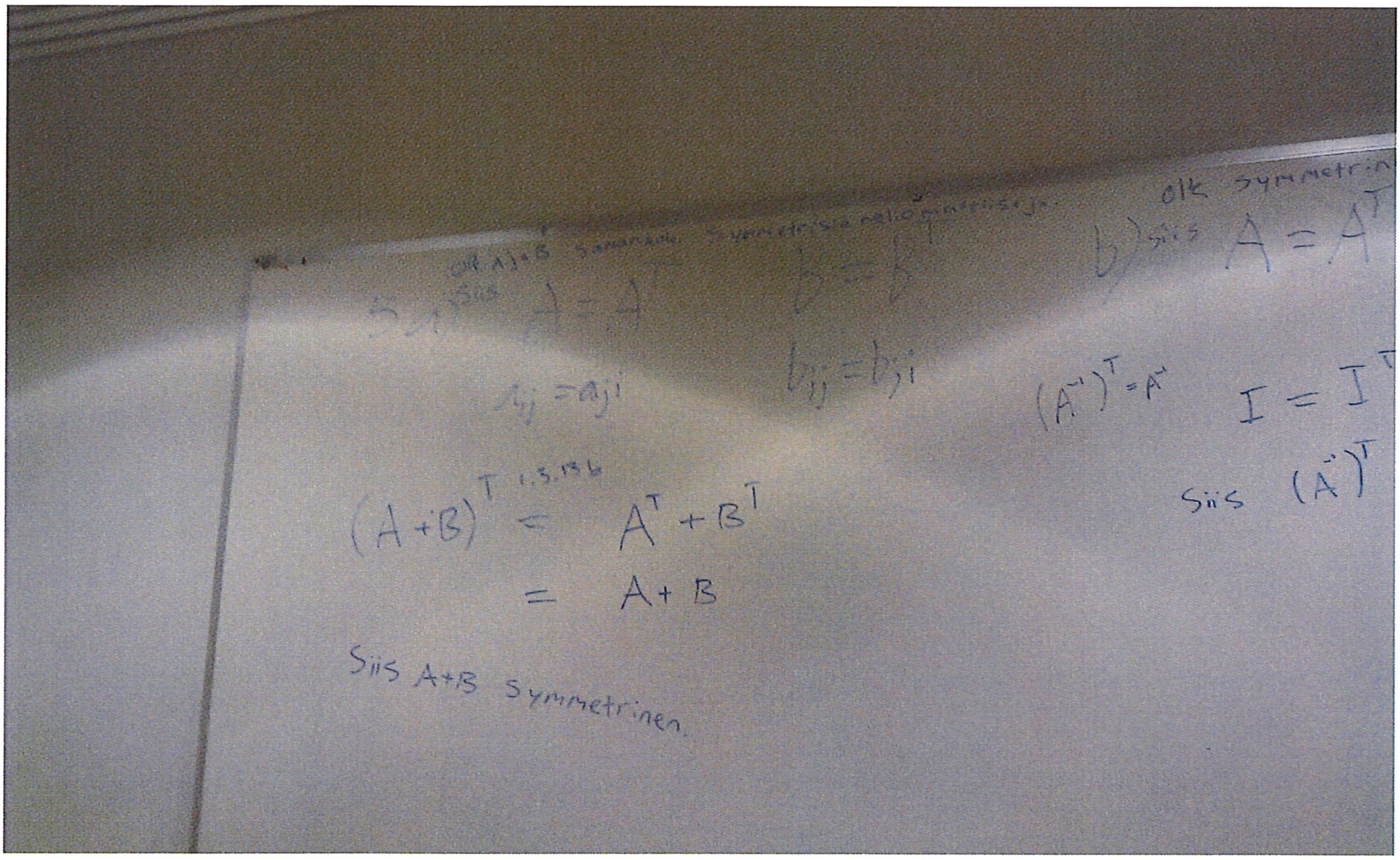
4) a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

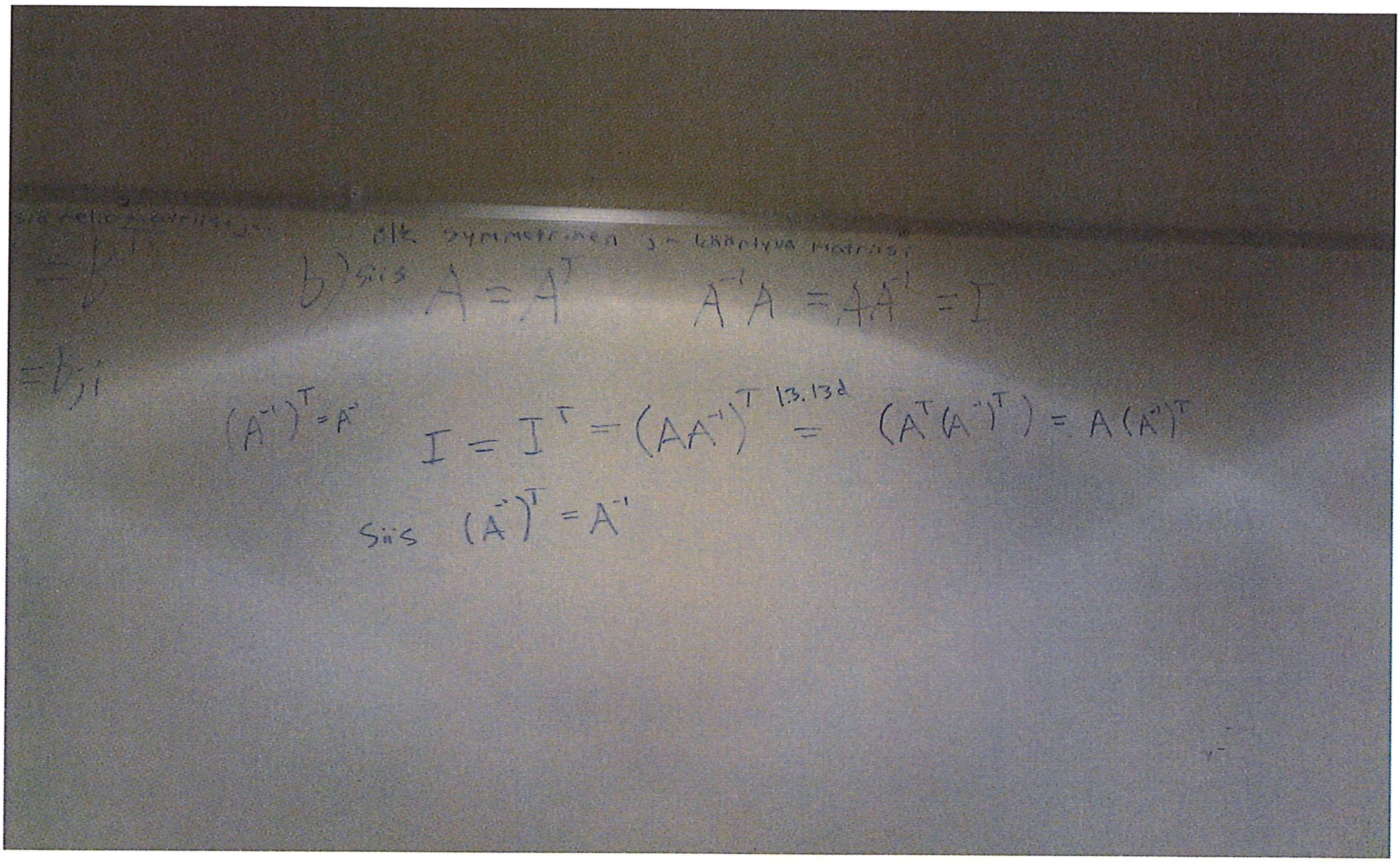
b) on oikein

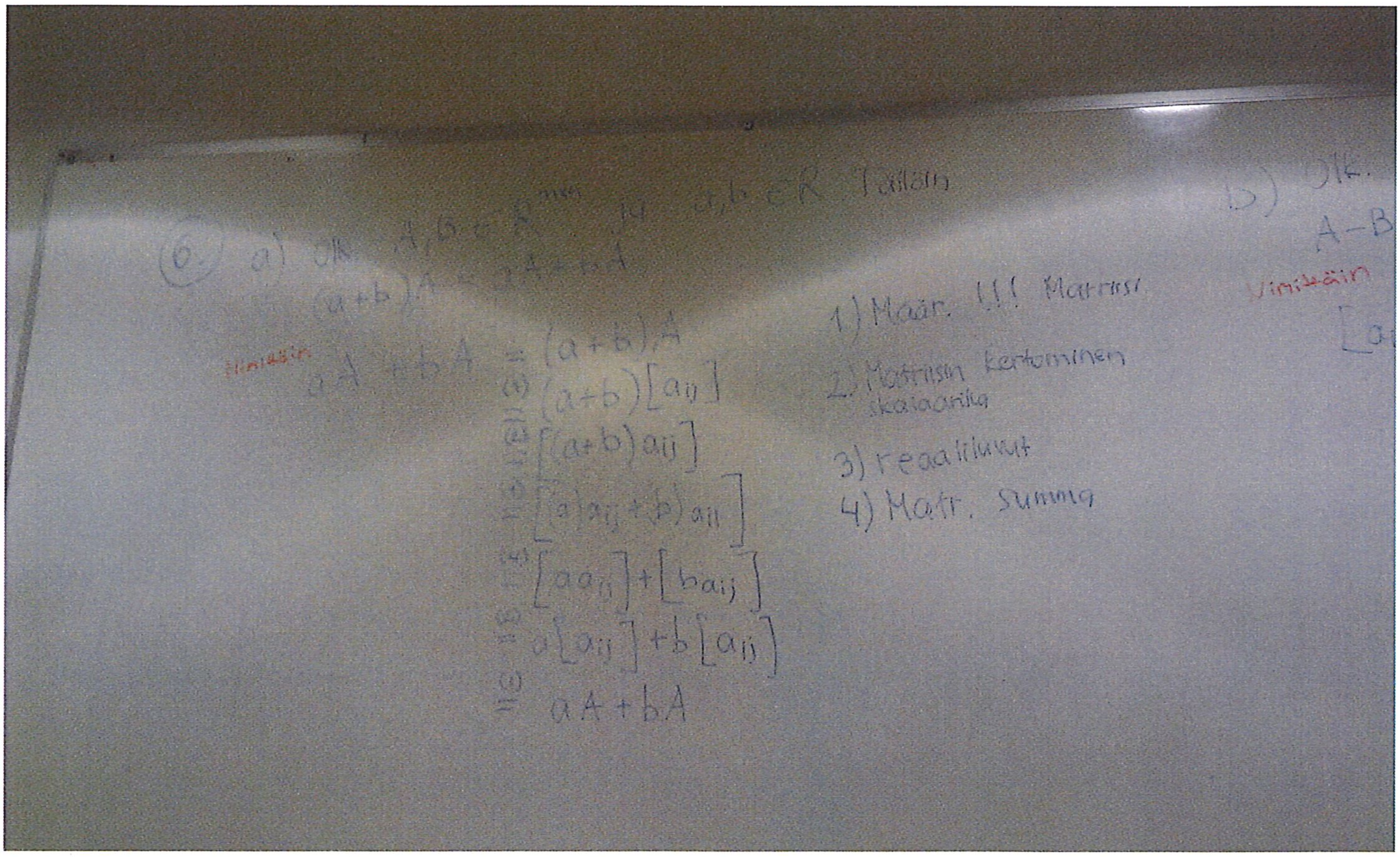
$(A+B)(C+D)$ osittelulait

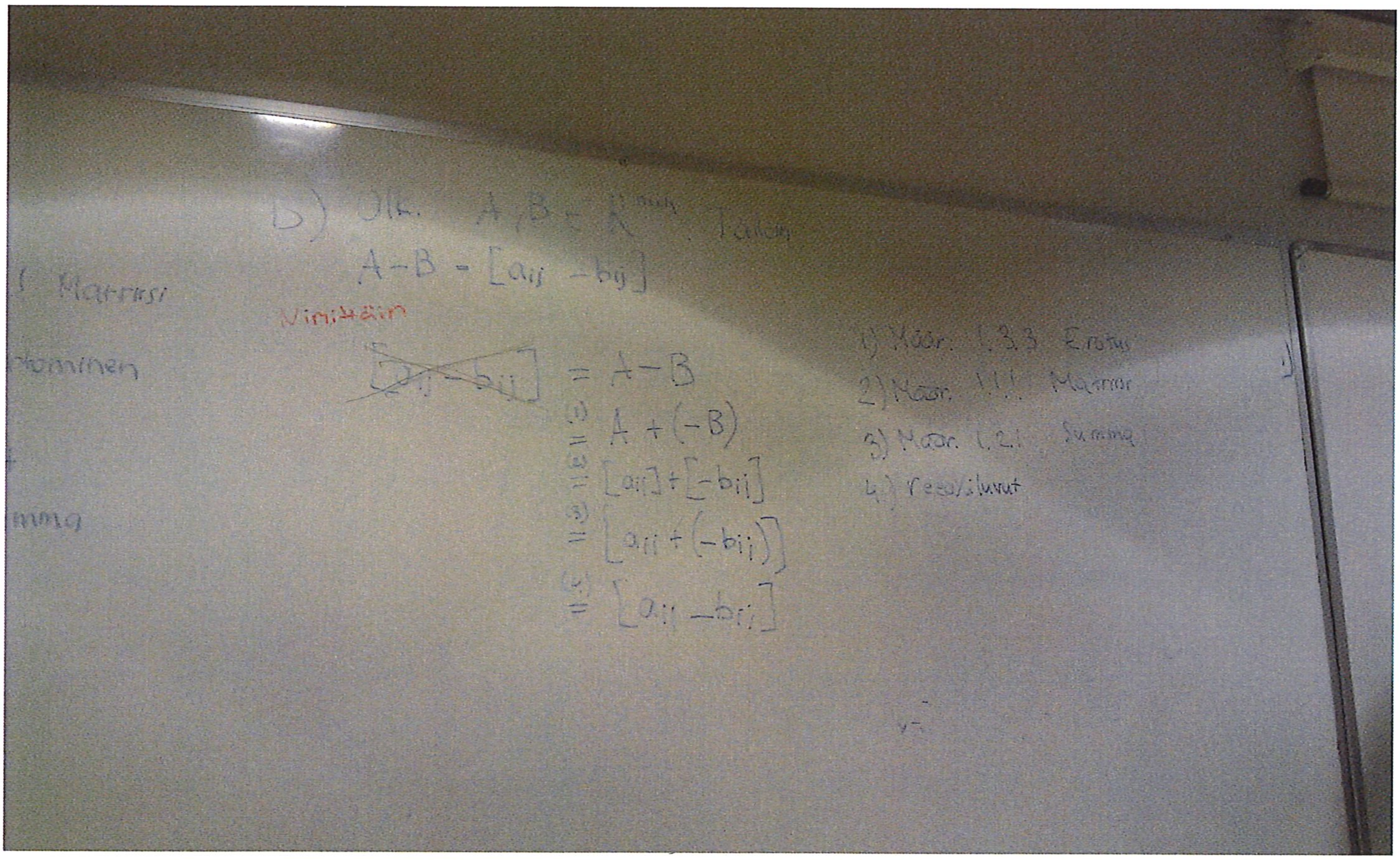
$= A(C+D) + B(C+D)$ — " —

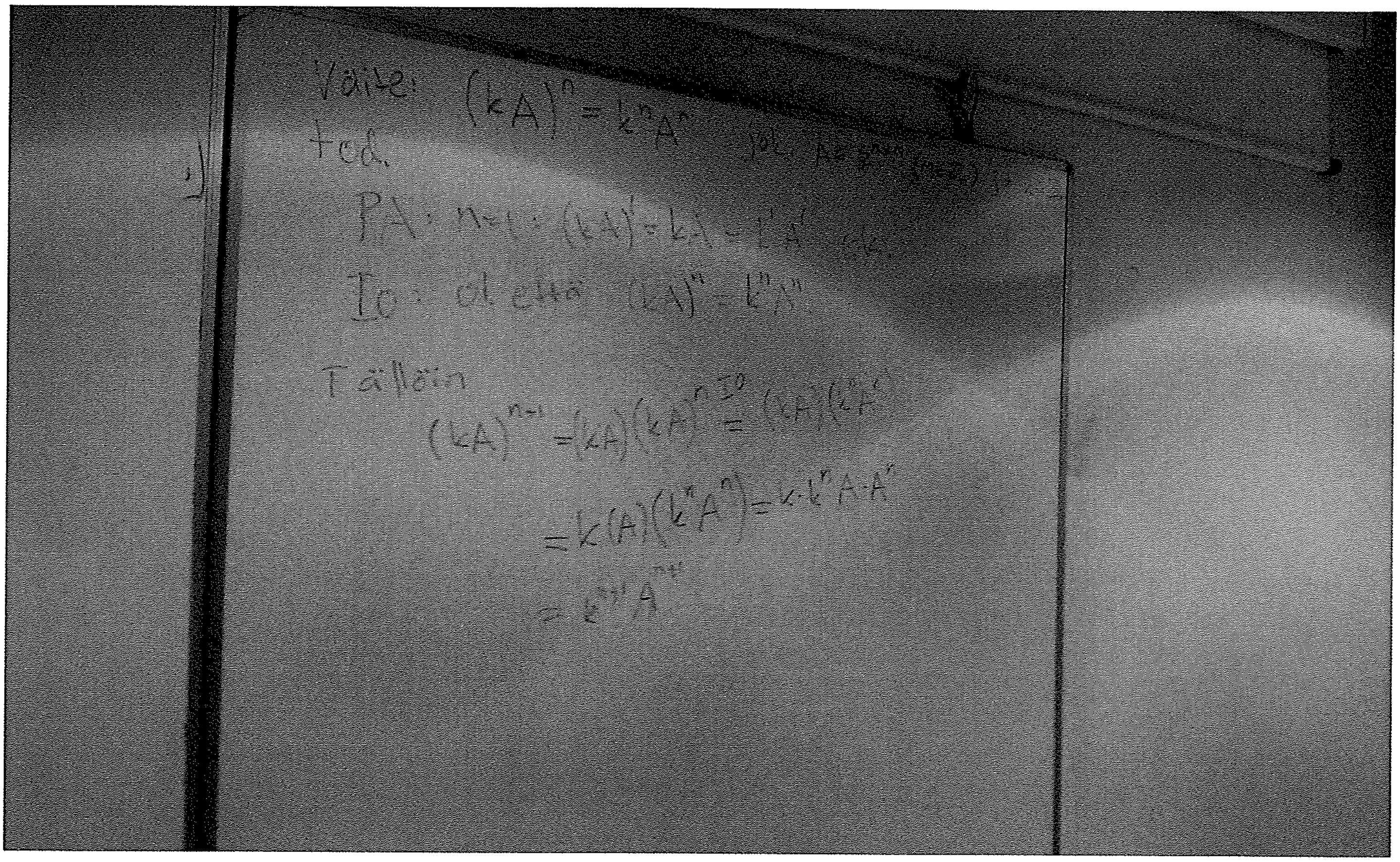
$= AC + AD + BC + BD$

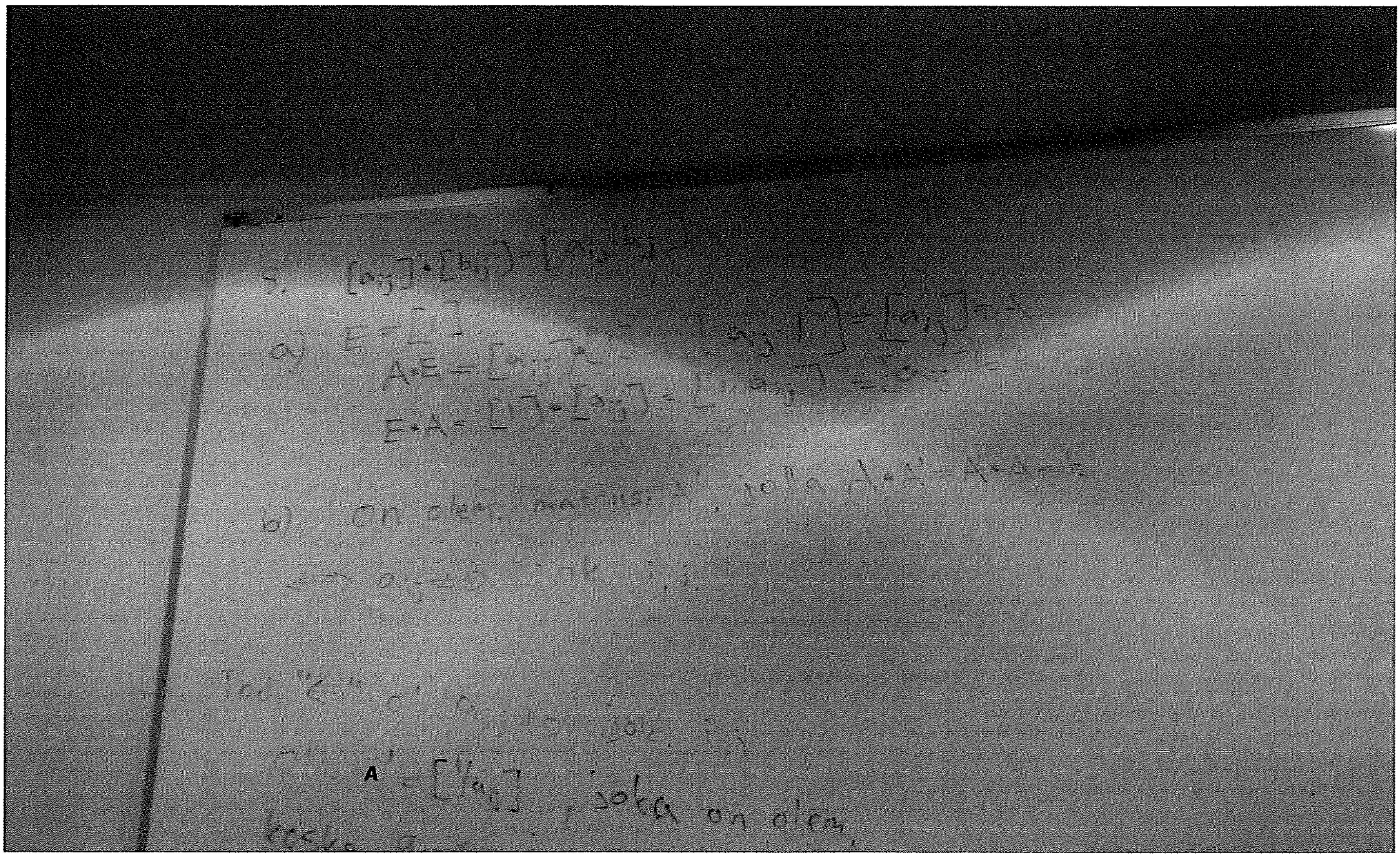

(A+B)^T = A^T + B^T
= A + B
Siis A+B symmetrinen.
(A^-1)^T = A^-1
I = I^T
Siis (A^-1)^T

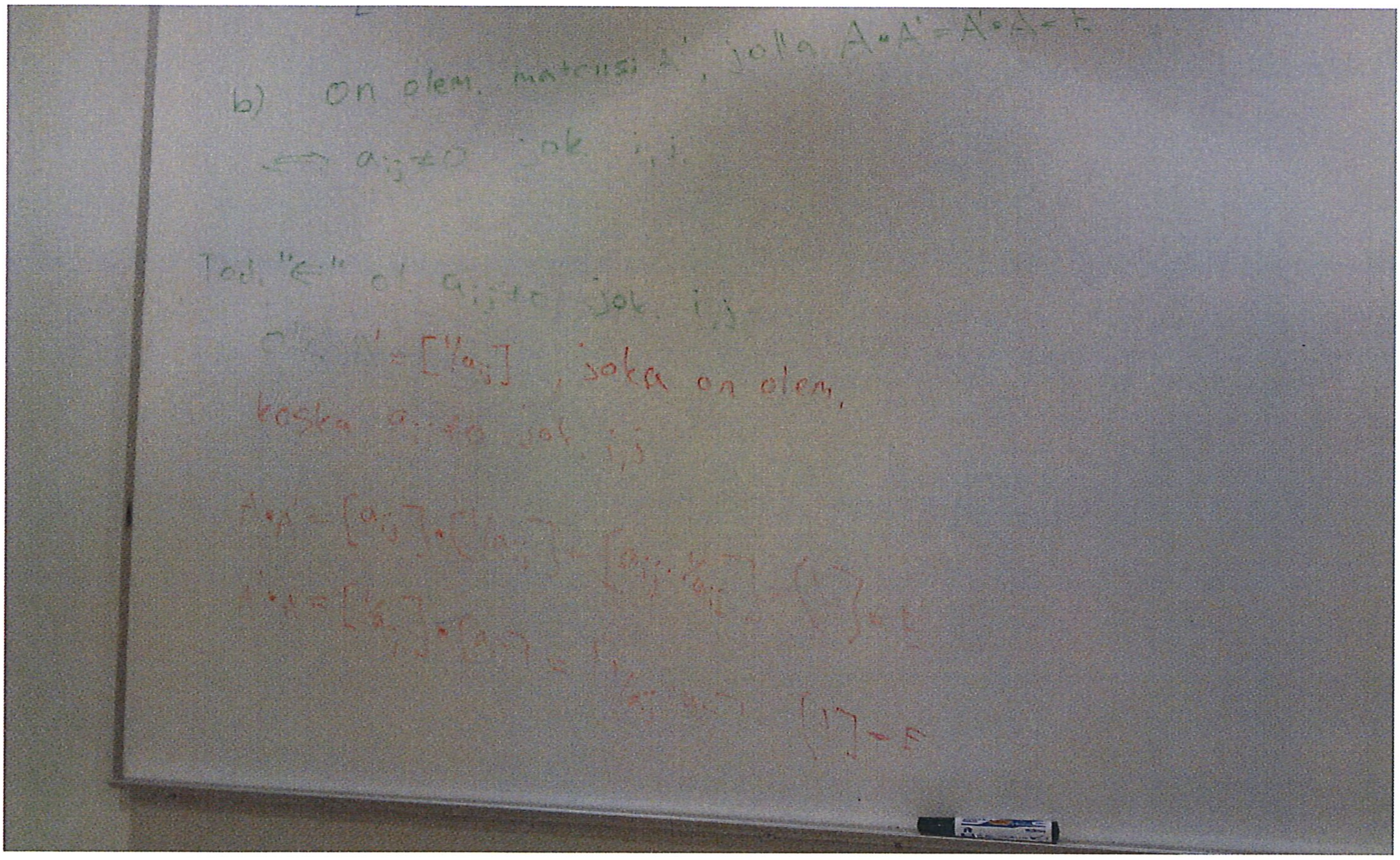

olk. symmetrinen ja kääntyvä matriisi
b) Siis $A = A^T$ $A^{-1}A = AA^{-1} = I$
$(A^{-1})^T = A^{-1}$
$I = I^T = (AA^{-1})^T \overset{13.13d}{=} (A^T(A^{-1})^T) = A(A^{-1})^T$
Siis $(A^{-1})^T = A^{-1}$

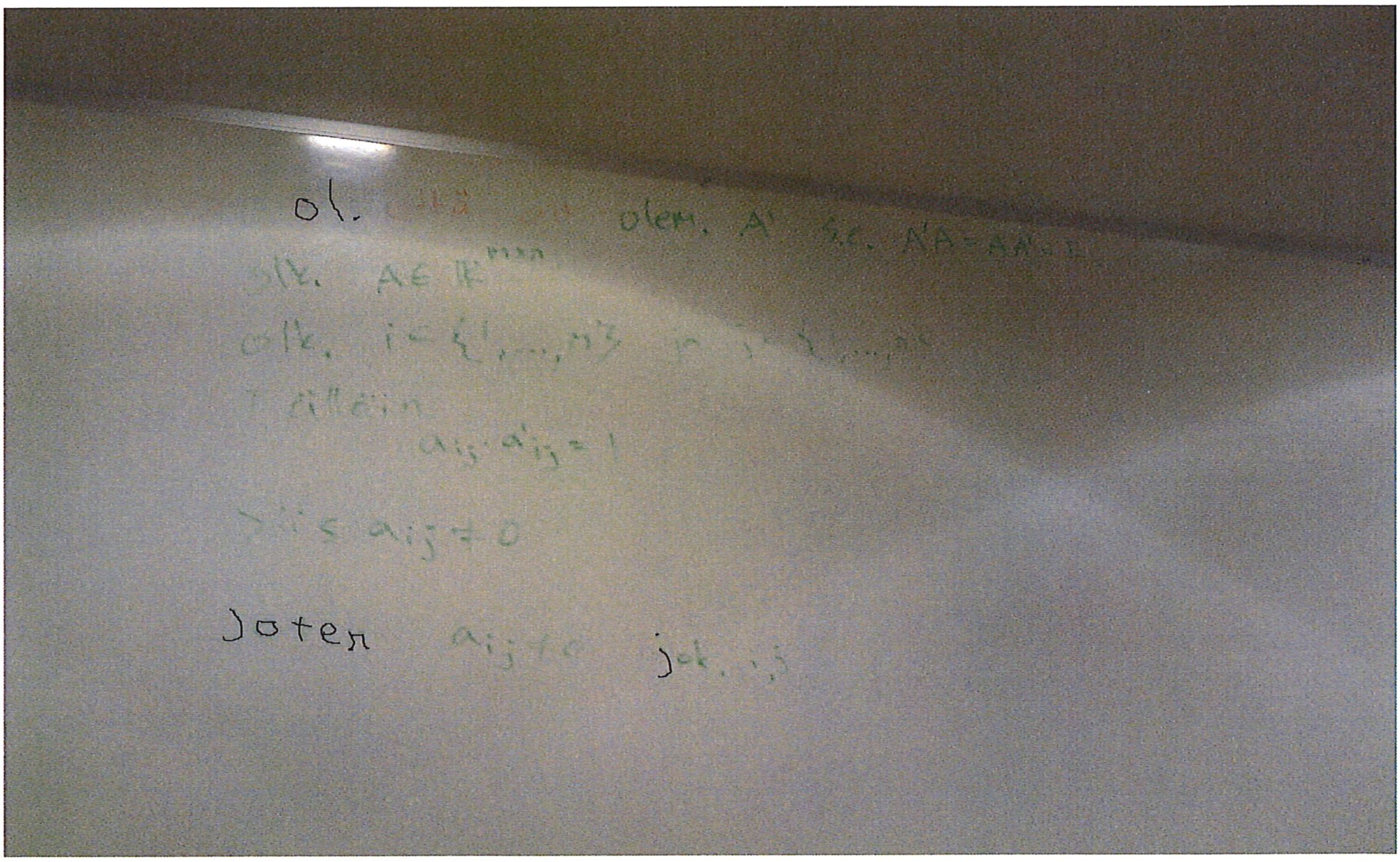

6. a) Olk. A, B ∈ R^{m×n} ja a, b ∈ R. Täällä
(a+b)A = aA + bA
aA + bA = (a+b)A
= (a+b)[aij]
= [(a+b)aij]
= [aaij + baij]
= [aaij] + [baij]
= a[aij] + b[aij]
= aA + bA
1) Määr. !!! Matriisi
2) Matriisin kertominen skalaarilla
3) reaaliluvut
4) Matr. summa
b) Olk.
A−B


Matriisi
Niinpä
A + (−B)
1) Määr. 1.3.3 Erotus
2) Määr. 1.1.1 Matriisi
3) Määr. 1.2.1 Summa
4) Reaaliluvut


Väite: $(kA)^n = k^n A^n$
tod.
PA: $n=1$: $(kA)^1 = kA = k^1A^1$
Io: ol. että $(kA)^n = k^nA^n$
Tällöin
$(kA)^{n+1} = (kA)(kA)^n \overset{Io}{=} (kA)(k^nA^n)$
$= k(A)(k^nA^n) = k \cdot k^n A \cdot A^n$
$= k^{n+1}A^{n+1}$


a)
b) On olem. matriisi
A
, joka on olem,


b) On olem. matriisi A', jolla
, joka on olem.


ol. olem. A' s.e. A'A = AA' = I
olk. A ∈ ℝ^{n×n}
olk. i ∈ {1, ..., n}
Silloin
a_ij · a'_ij = 1
Siis a_ij ≠ 0
Joten a_ij ≠ 0 jok. i, j